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# Valuation Analysis of Convertible Bonds in China: Does Theoretical Value Deviate from Actual Market Price?

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**Abstract**

This study compares the theoretical valuation with the actual price of 30 convertible bonds from the Chinese market in 2019. Based on Black-Scholes Model, Cox, Ross and Rubinstein model and Monte Carlo simulation approach, our result shows: First, the stepwise regression and Monte Carlo simulation are closest to the actual market price, with the lowest deviation rate of 0.078% and -9.09%, respectively. Second, this paper also analyses the deviation between the theoretical and the actual market price. It is found that maturity is an essential factor affecting the value of convertible bonds, and it is the most underestimated on the first day of issuance. However, the degree of underestimation gradually diminishes as the life span of the bond decreases over time. Third, the deviation between the actual and theoretical price is that valuation models occur as the models seldom consider embedded call options, sell-back terms and redemption terms. As a policy implication, the price discrepancies between theoretical and actual prices should be monitored

continuously by the regulators and market practitioners in China's financial markets.

***Keywords:*** *convertible bonds, embedded options, Black-Scholes, Cox, Ross and Rubinstein model, Monte Carlo simulation*

## 1. Introduction

In 1874, the Rome, Watertown and Ogdensburgh Rail Road Company issued convertible bonds to the public for the first time, making the railroad project a success. The lower financing cost of convertible bonds and embedded conversion options make it an innovation in the capital market. Furthermore, with the emergence of option pricing models in the 1970s, convertible bonds have become an important financing tool. Over the years, the market size has steadily increased as the United States, Japan, and the European Union actively promote their bond market (Thoma, 2015).

In China's capital market, convertible bond, as a financial derivative product, has increasingly become a new financing tool to replace the company's fixed and revolving facilities from commercial banks. According to statistics from the Wind database, the size of China's equity financing and convertible bond financing in 2018 was 138.7 billion yuan and 79.4 billion yuan, respectively. The market size of convertible bonds in the first quarter of 2019 had exceeded 100 billion yuan, higher than the value reported in 2018. By the end of 2019, there were 151 convertible bonds with a total volume of 269.5 billion yuan.[1]

Looking back on the development of convertible bonds, Tian and Chen (2020) state there are four stages of development. From 1991 to 1997, convertible bonds were mainly issued by small companies during the first period. The first non-listed company that issued convertible bonds in 1991 was Qiong energy. This was followed by Shenzhen Baoan Enterprise and China Textile Machinery in 1992 and 1993, respectively. Only 13 Chinese companies raised funds through issuing convertible during this period.

The second stage from 1997 to 2001 could be termed the Exploratory stage. China Securities Regulatory Commission (CSRC) issued the Interim Measures for the Administration of Convertible Corporate Bonds. These measures specified the standards to reach for the issuance of convertible bonds. It also stated the commitments to be complied with by the underwriters.

The third stage from 2001 to 2009 was known as the maturing period. The CSRC issued measures for the issuance of convertible corporate bonds by listed companies in April 2001. These new measures filtered out small firms with weak performance from issuing convertible bonds and contributed to substantial market size growth. By the end of 2009, 60 listed firms issued convertible bonds in Shenzhen and Shanghai stock exchanges. As reported, the market continued to grow, and by the end of 2017, the convertible bonds reached a total volume of 60.3 billion yuan (Tian and Chen, 2020).

This study fills the lacuna between research and practice. An analysis of convertible bonds' pricing model is needed to understand the market practice better. It is clear from the literature that the research on convertible bonds' valuation has lagged in China. First, this study utilizes the stepwise regression model, Black-Scholes Model (1973), Cox, Ross and Rubinstein Model (1979) and Monte Carlo simulation approach to compare and select the most suitable valuation model for China's convertible bond market. Second, this study will attempt to ascertain the reasons for the deviation of the actual price from the valuation model.

This paper is organized as follows. Section two reviews the literature. Section three elaborates the data dan methodology. Section four discusses the results. The last section discusses and reasons for the deviation between actual and theoretical models and concludes the study with three key findings.

## 2. Literature Review

### *2.1. Black-Scholes Model*

The Black-Scholes (1973) model is the first option pricing model published in the *Journal of Political Economy*. In the same year, Merton (1973) published the article "Theory of rational option pricing" to further extend the Black-Scholes formula's assumptions. In the article "On the Pricing of Corporate Debt", Merton (1974) used the option pricing model to solve enterprises' pricing problems (Haque, Topal and Lilford, 2014).

Numerous scholars have also converted convertible bonds based on the B-S option valuation model (Kremer and Nyborg, 2004; Tsiveriotis and Fernandes, 1998). It is believed that the B-S model has good valuation ability for convertible bonds and that there is a positive correlation between its convertible bond price and its stock price.

In a later study, Kimura (2017) identified the advantage of the B-S model. The analytic expression of hedging parameters and leverage effects are identified to provide more definitive conclusions for derivative asset trading strategies.

### *2.2. Cox, Ross and Rubinstein Model*

Ingersoll (1977) and Brennan and Schwartz (1977) first proposed the single-factor model for convertible bond valuation. This procedure introduces firm value into the Black-Scholes option valuation model, assuming that the firm does not consider dividends, deducing the partial differential equation of the value of convertible bond options.

Brennan and Schwartz (1980) consider adding the term structure of interest rates to the valuation model of convertible bonds, thereby establishing a two-factor model based on company value and interest rates. Considering the issue of dividend payment and fixed coupon interest payment, using the Vasicek interest rate model (Vasicek, 1977), the partial differential equation that the theoretical value of convertible bonds is satisfied by the company value and interest rate fluctuations are derived.

However, this valuation model does not consider the impact of redemption clauses. Vasicek model may have a negative interest rate. Athanassakos and Carayannopouslos (2000) replace the random interest rate model in the Brennan and Schwartz (1980) valuation model with the CIR model (Cox, Ingersoll and Ross, 1985) and assume that other conditions will not change.

Another strand of literature focuses on the binary tree model for interest rates and stock prices (Hung and Wang, 2002; Chambers and Lu, 2007; Yang and Yi, 2010; Xu, 2011; Atilgan, Bali and Demirtas, 2013; Huang, Liu and Rao, 2013; Grinderslev and Kristiansen, 2017).

In another study, Lyuu, Wen and Wu (2014) focus on the two factors of log stock prices and log interest rates when evaluating convertible bonds. They propose a corresponding binary tree valuation, a reasonable probability model to value interest-sensitive financial products. Bechmann, Lunde and Zebedee (2014) value convertible bonds through the nature of real and imaginary options and take into account the announcement effect.

Later work by Okimoto and Takaoka (2017) examined a two-factor model based on stock prices. This convertible bond valuation model can decompose convertible bonds' value into investment value, warrant value, and issuer's redemption option value. Lu and Xu (2017) introduce the two-factor willow tree method. They assumed that the interest rate obeyed the logarithmic Ho-Lee model. The stock price follows the geometric Brownian motion. Liebenberg, van Vuuren and Heymans (2017) point out that credit risk is not considered in the valuation of convertible bonds, so the theoretical value and the actual price of convertible bonds. A valuation model of convertible bonds with credit risk was established.

Batten, Khaw and Young (2018) use a numerical simulation method to test the five influencing factors of the value of convertible bonds and concluded that the most significant factor affecting convertible bonds is the enterprise's market value. They also proposed a discrete-time model of arbitrage-free pricing of convertible bonds. In another study, Park, Jung and Lee (2018) derived the optimal conversion strategy for bond investors and optimal redemption strategy for bond issuers. They decompose the value of convertible bonds into ordinary bond value plus warrant value minus redemption option value, that is

Convertible bond value = ordinary bond value + warrant value
– redemption value

### *2.3. Monte Carlo Simulation Approach Research*

Carayannopoulos and Kalimipalli (2003) also regard the redemption clause as an option, thereby splitting the value of convertible bonds into bond value and equity value and considering the default risk as a variable following the random fluctuation of stock prices, using the finite-difference method to solve. Lau and Kwok (2004) believe that convertible bonds contain a lot of option value. The complexity of option value determines that the analytical solution of convertible bond value is challenging to obtain. Therefore, combining the Monte Carlo simulation approach and finite-difference method and considering the impact of additional clauses on the valuation model has proposed a more accurate valuation of US convertible bonds.

Yigitbasioglu *et al.* (2004) further examine the two-factor model based on stock price and the interest rate based on Ayache *et al.* (2003). The Monte Carlo simulation approach is more efficient than the finite-difference and binary tree models. Kimura and Shinohara (2006) assume that the underlying stock has no dividend payment. The issuer has credit risk, a theoretical model, and a Monte Carlo simulation to establish a convertible bond valuation model that is not redeemable with modified terms and a closed-form analytical formula.

Several researchers have also used Monte Carlo simulation to value the bond price (Chambers and Lu, 2007; Ammann, Kind and Wilde, 2003 and 2008; Moreno, Navas and Todeschini, 2009; Li, 2010; Bielecki *et al.*, 2011; Beveridge and Joshi, 2011). Batten, Khaw and Young (2014) use option and bond pricing technology, consider credit risk and redemption terms and propose a direct and straightforward valuation model. The mathematical derivation results show a linear relationship between credit risk and redemption terms. Dubrov (2015) proposes a new stochastic algorithm method based on machine learning, which can price American options and convertible bonds using Monte Carlo simulation.

### *2.4. Hybrid Nature Of Convertible Bonds*

According to the issuance terms, convertible bonds (abbreviated as CB) are company bonds issued by companies under legal procedures, allowing investors to convert them into a certain number of shares within a specified period. Convertible bonds are financial products, usually with the following characteristics. The holders can be converted into ordinary shares or equivalent cash of the issuing company at an agreed price. It brings additional value to holders by providing conversion rights for upside potential, while on the other hand, issuers benefit from reduced interest rates. If the bondholder chooses to convert during the bond duration, the bond will be redeemed, and the holder will receive some common shares from the issuer.

As long as the bondholder does not convert the bond, he will regularly receive coupons and repay the principal at maturity. The hybrid nature has inspired some models to consider the convertible bond value composed of a bond component and an option on the stock (Ou, 2010). If the convertible bond remains live till maturity, the payoff at maturity is

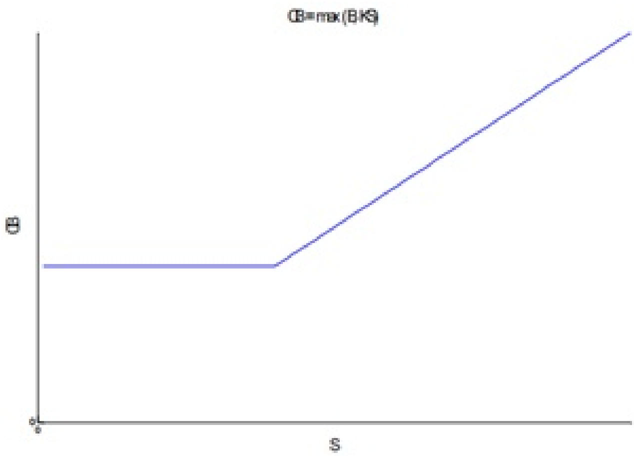
$$CB = \max(B, kS) = B + (kS - B)^{+} \tag{1}$$

where $B$ = Bond price

$k$ = Strike price

$S$ = Stock price

**Figure 1** The Payoff of a Convertible Bond with k=1 as the Conversion Ratio



It becomes clear that convertible bonds are a hybrid financial product with bonds and stocks (shown in Figure 1). The underlying risks of convertible bonds come from the stock price and interest rate variation. The hybrid nature has inspired some models to consider the convertible bond value. It can be composed of a bond component and an option on the stock.

### *2.5. Callable and Puttable Features of Convertible Bonds*

The put provision allows the holder to return the convertible bond to the issuer at a certain point in time in exchange for a predetermined amount of cash, thus providing downside protection in the event of rising interest rates (Homaifar and Zhao, 2019).

When the convertible bond is callable, the issuer can repurchase the bond at a predetermined strike price, which often changes during the bond's duration. Therefore, redemption clauses are usually used to force the early conversion of bonds.

The Early conversion of a convertible bond is not optimal for the holder under certain conditions; hence, the redemption clause reduces convertible bonds' value. If interest rates fall or stock prices rise, it will limit investors' returns (Jarrow, Liu and Wu, 2010). All convertible bonds are callable bonds in this paper, and puttable bonds are not considered.

### *2.6. Convertible Bonds Market in China*

Although the convertible bond market was in the maturing stage from 2001 to 2009, the liquidity of the Chinese corporate bond market as measured by the trading volume was low (Xiao, 2002). In addition, Zhu (2009) states that China's convertible bonds market faces several problems such as public issues, innovation in product design and credit rating system. Yan (2011) also reports some regulations in China that require the issuing firms to have continuous profits in the past three years, with an average profit of 10 percent.

In addition, numerous authors study the pricing of convertible bonds. Liao *et al.* (2012) claim that the issuance of convertible bonds causes higher equity risks, leading to greater volatility of stock returns. They conclude the negative effect of stock delusion on convertible bond prices. In another study by Huang and Feng (2017), they propose a classic arbitrage strategy of constructing a market-neutral portfolio consisting of the extended position of convertible bonds and the short position of underlying stocks.

## 3. Methodology

### *3.1. Data*

**Table 1** Selected Convertible Bonds

| Convertible bond code | Convertible bond name | Convertible bond code | Convertible bond name | Convertible bond code | Convertible bond name |
|---|---|---|---|---|---|
| 128080.SZ | SF | 128083.SZ | XB | 128081.SZ | HL |
| 113550.SH | CQ | 113545.SH | JN | 113542.SH | HK |
| 123034.SZ | TG | 128075.SZ | YD | 128069.SZ | HS |
| 110059.SH | PF | 113544.SH | TL | 113536.SH | SX |
| 127014.SZ | BF | 123032.SZ | WL | 113534.SH | DS |
| 128078.SZ | TJ | 123030.SZ | JZ | 113024.SH | HJ |
| 128077.SZ | HX | 113543.SH | OP | 128062.SZ | YY |
| 110057.SH | XD | 123025.SZ | JC | 128059.SZ | SY |
| 110056.SH | HT | 113022.SH | ZS | 127011.SZ | ZD |
| 113528.SH | CC | 110051.SH | ZT | 128056.SZ | JF |

SH denotes Shanghai Stock Exchange
SZ denotes Shenzhen Stock Exchange

Source: Wind database.

As shown in Table 1, this paper selects 30 convertible bonds listed in 2019 as its sample. The convertible bond code is marked with SZ or SH at its back. SZ stands for Shenzhen Stock Exchange. SH stands for the Shanghai Stock Exchange. The data are obtained from each converted bond issuance announcement, the bond issuing company's annual report, and the underlying stocks' transaction records.

### *3.2. Convertible Bond Valuing Model*

#### *3.2.1. Stepwise regression model*

Zhao (1994) proposes and proves in theory that the stepwise regression algorithm could be extended from static systems to other economic studies. Stepwise regression is the gradual screening of independent variables, and the screening process is in and out. In the beginning, the dependent variable and each white variable are subjected to a one-variable regression, and the one most closely related to y is selected to test the most significant one-variable linear regression equation. Then introduce the second variable to the model. The principle is that it has a sizeable F-test value than other variables entering the model. If it is not significant, it is eliminated.

When a variable is introduced, it is tested to decide whether it can be excluded from the model. In this way, this step is repeated until no new variable can be introduced, and no old variable can be eliminated, the regression equation is finally established.

#### *3.2.2. Black-Scholes-Merton valuing model*

Black and Scholes (1973) and Merton (1973) introduce the partial differential equations into the field of option pricing based on risk-free arbitrage and hedging principles, solve the contingent pricing methods, and derive the Black-Scholes-Merton Partial differential equation.

This formula has become the cornerstone of modern financial derivatives pricing and has been widely used in theory and practice. The risk-free arbitrage pricing method is brand new and leapfrogging in option pricing. The practice has proved that the model has become the key to developing and innovating derivative financial instruments. The model's assumptions are as follows:

1. The price of the underlying asset is subject to random fluctuations and follows a log-normal distribution;
2. The continuous risk-free rate is known and constant;
3. The volatility of the underlying asset is known and constant;
4. The market is frictionless;
5. There are no cash flows on the underlying asset;
6. The options valued are European-styled options.

Based on the above assumptions and the main relevant factors affecting the value of options, Black-Scholes-Merton's pricing model:

$$C(t) = SN(d_1) - Xe^{-rT}N(d_2)$$
$$d_1 = \frac{\ln(S/X) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln(S/X) + (r - 0.5\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad (2)$$

where:
*C*: the price of the bond option;
*S*: the current market price of the underlying stock;
*r*: risk-free interest rate (as continuous compound interest rate);
*σ*: underlying stock price volatility
*T*: expiry date
*t*: current time
*X*: conversion price
*N*(*d*): cumulative probability distribution function of a standard normal distribution variable.

Given the above variables, the Black-Scholes-Merton model is used to calculate the call option's value.

### *3.2.3. Cox, Ross and Rubinstein model (CRR model)*

Cox, Ross and Rubinstein (1979) propose another method for pricing options in the risk-neutral world, the Binomial Tree model, which has become an essential method for pricing financial derivatives. This method assumes only two possible stock price outcomes after the time interval Δt. It then uses the branches to show the path and combines the expectation theory to describe the underlying asset and option prices.

The assumption of using the binary tree model is similar to those of the BSM model, but the binary tree model's assumptions can be relaxed to varying degrees. The most crucial difference between the two models is that the binary tree model assumes that the underlying asset price state process is infinitely divisible.

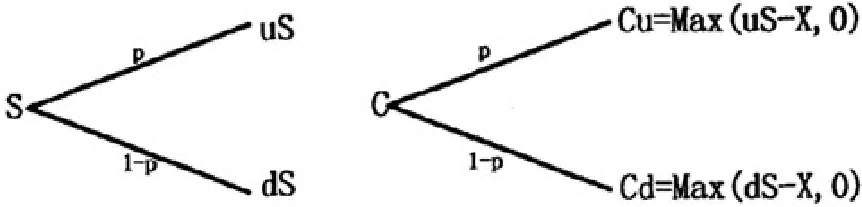
In the binary tree model, the time interval between price changes is set to small-time interval Δt, and it is assumed that there are only two ways in which the price can move after the time interval Δt. Take the initial state as an example: after the time interval Δt, the price of the underlying stock may rise to u times $S_0$ with a probability of P; it may also drop to d times $S_0$, where u≥1, d≤1, u * d = 1, as shown in Figure 2. According to the option value principle, the option value will also be different when stock prices are different. Hence, they are $C_u$ and $C_d$. The formula is

$$C = e^{-rf\Delta t} [pC_u + (1 - p)C_d] \quad (3)$$

where:

$p = \dfrac{e^{-r_f\Delta t} - d}{u - d}$ is the risk-neutral probability,
$r_f$ is the risk-free interest rate,
$C$ is the callable bond price,
$C_u = \text{Max}(uS–X,0)$,
$C_d = \text{Max}(dS–X,0)$.

**Figure 2** Relationship between Asset Price and Option Value Status during Δt time interval




*3.2.4. Monte Carlo Simulation approach*

Monte Carlo simulation is a numerical method for European options. The basic idea of this method is that given the stock price's distribution function and parameters, the option's validity period is divided into N time intervals.

With the help of a computer, each time interval can be randomly sampled from a standard normal distribution. The change of the stock price and possible stock price movement is used to obtain the return of its stock option at T.

This step is repeated 10,000 times to obtain the set of option prices at time T and perform a simple arithmetic average of 10,000 random samples to obtain the expected return of options at time t.

Using the concept of risk-neutral pricing, the option's price at the current time can be obtained by discounting the option $C_i$ at the future T at a risk-free rate (Sowey, 1973). The formula is

$$dS = rSd_t + \sigma SdW_t \tag{4}$$

W is Standard Brown Motion.

After the stock price simulation is completed, the option value can be calculated at different times. According to the redemption strategy, the conversion and resale strategy of the convertible bonds set the corresponding boundary conditions to determine the convertible bonds' value.

### *3.3. Stepwise Regression Model*

The convertible bond valuation model uses the price of the first day of listing the convertible bond as the dependent variable, various factors that affect the price as the independent variable, and a stepwise regression model to obtain the convertible bond valuation model. In the analysis below, the stepwise regression model results will be compared with the three models, namely the B-S model, CRR model and Monte Carlo simulation, to select the most suitable valuation model for the Chinese financial market.

#### *3.3.1. Independent and dependent variables*

Five independent variables are selected in the stepwise regression model for the first day of convertible bonds valuation. The independent variables are stock price (SP), the increase and decrease of price (ID), the initial conversion price (CP), conversion ratio (CR), the conversion premium rate (CPR) in percentage. The dependent variable is the convertible bond price (CBP) on the first day of listing. The model is as follows:

$$CBP = b_1SP + b_2ID + b_3CP + b_4CR + b_5CPR + C + \varepsilon \qquad (5)$$

*3.3.2. Visual Basic for Applications (VBA)*

Visual Basic for Applications (VBA) is a macro language of Visual Basic. More precisely, it is an automation language that can automate commonly used programs and create custom solutions. It can quickly process financial data in the financial modelling process, so it is widely used in the financial industry. The B-S model's model results, the CRR model and the Monte Carlo simulation are all obtained by VBA. Bartolacci, LeBlanc, Kayikci and Grossman (2012) show that VBA can solve complex financial models accurately in their work.

## 4. Results

### *4.1. Descriptive Statistics*

**Table 2** Descriptive Statistics of Variables

| Variables | N | Mean | Max | Min | Std. Dev | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| SP | 30 | 21.38 | 117.00 | 5.54 | 25.20 | 2.54 | 9.10 |
| ID | 30 | 1.59 | 19.18 | 0.07 | 3.42 | 4.76 | 25.04 |
| CP | 30 | 21.25 | 101.46 | 5.70 | 21.90 | 2.33 | 8.05 |
| CR | 30 | 7.99 | 17.54 | 0.99 | 4.48 | 0.38 | 2.56 |
| CPR | 30 | 22.98 | 127.00 | -6.00 | 23.19 | 3.11 | 14.66 |
| CBP | 30 | 109.12 | 123.64 | 97.00 | 6.61 | 0.28 | 2.25 |

N = observations.

From Table 2, the mean value of the dependent variable's convertible bond price is 109.12. The standard deviation is 6.61, indicating that the variable has a small range of variation. Convertible bond prices are also very close, with minor fluctuations, the skewness is 0.28, and the skewness is towards the right side.

Besides, all variables skew towards the right side. Except for the conversion ratio and the convertible bond price, the kurtosis of other variables is greater than 3, which indicates that the distribution of most variables is steeper than the normal distribution and deviates from the normal distribution, among which the increase and decrease deviate from the normal distribution most obviously.

### *4.2. Correlation Analysis*

**Table 3** Correlation Analysis of Variables

| | SP | ID | CP | CR | CPR | CBP |
|---|---|---|---|---|---|---|
| SP | 1 | | | 0.143 | -0.13 | -0.147 |
| ID | .442* | 1 | | | -0.197 | -.446* |
| CP | -0.16 | -0.037 | 1 | .805** | 0.263 | .648** |
| CR | 0.143 | 0.107 | .805** | 1 | 0.007 | .554** |
| CPR | -0.13 | -0.197 | 0.263 | 0.007 | 1 | .557** |
| CBP | -0.147 | -.446* | .648** | .554** | .557** | 1 |

The asterisks * and ** denote statistical significance at 5% and 1%, respectively.

The correlation can reflect the quantitative relationship between variables to a certain extent. Through Table 3, convertible bond price and stock price, the increase and decrease (%), conversion price, conversion ratio, and the conversion premium rate (%) are at the level of 5% or 1% significantly related. Moreover, it is found that the increase and decrease (%) and the initial conversion price and the conversion premium rate are significantly related, so we can consider stepwise regression screening variables to obtain variables that affect the conversion value.

As shown in Table 4, our results show that four variables ultimately affect the convertible bond value through the stepwise regression method: the stock price, the increase and decrease, the conversion premium rate, and the conversion ratio. The four variables are significant at the 5% level. The estimated value of the stock price coefficient is 0.20, indicating that for every additional unit, the convertible bond price will increase by 0.20. For every 1% increase in the conversion ratio, the convertible bond price increases by 0.25. For every increase and decrease of 1 unit, the convertible bond value will increase by 0.63.

### *4.3. Stepwise Regression Model Analysis*

**Table 4** Stepwise Regression Model Analysis

| Model | Variable | Coefficient | Std Error | t-Statistic | P-value | |
|---|---|---|---|---|---|---|
| 1 | **C** | 104.77 | 1.40 | 74.63 | 0.00 | |
| | **SP** | 0.16 | 0.04 | 4.04 | 0.00 | |
| | **ID** | 0.59 | 0.29 | 2.05 | 0.05 | |
| | **R-squared** | | | | | 0.41 |
| | **Ad R-sq** | | | | | 0.32 |
| | **F-statistic** | | | | | 9.22 |
| 2 | **C** | 100.56 | 4.24 | 23.70 | 0.00 | |
| | **SP** | 0.20 | 0.06 | 3.43 | 0.00 | |
| | **ID** | 0.63 | 0.29 | 2.15 | 0.04 | |
| | **CPR** | 0.05 | 0.05 | 1.16 | 0.26 | |
| | **CR** | 0.25 | 0.32 | 0.79 | 0.44 | |
| | **R-squared** | | | | | 0.44 |
| | **Adj R-sq** | | | | | 0.35 |
| | **F-statistic** | | | | | 4.93 |

The level of significance is at 5%.
SP denotes Stock Price; CR denotes Conversion Ratio; ID denotes increase and decrease of Price; CPR denotes Conversion Premium Rate; C denotes Constant. The dependent variable is the convertible bond price (CBP).

However, from the result, the Adjusted R-squared is only 44%; the gap between the ideal situation is large, which shows that the model's explanatory ability is not strong. This phenomenon is that the sample size is small, only 30, and it may miss another important influencing factor in the model, which has affected the accuracy of model valuation.

According to the results of Table 4, we obtain a stepwise regression model of the first day of valuation of convertible bonds:

$$CBP = 100.56 + 0.20SP + 0.63ID + 0.05CPR + 0.25CR \quad (6)$$

**Figure 3** Comparison of Stepwise Regression Model Price with Actual Market Price

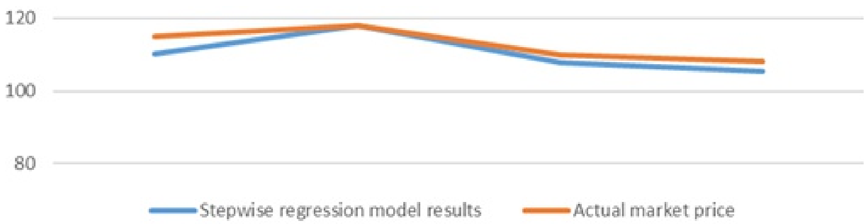



Table 5 is used to build the line in Figure 3. It can be seen from Figure 3 that the price of convertible bonds in the stepwise regression model is very close to the actual market price, and the deviation rate is only 0.00078, which shows that the variables selected in the model are valid. These variables have an impact on the price of convertible bonds.

According to Figure 3, the stepwise regression model's price is lower than the actual market price, indicating that convertible bonds' current market price is likely to be overestimated.

**Table 5** Stepwise Regression Model Results

| Convertible bond code | Stepwise regression model results | Actual market price | Deviation rate |
|---|---|---|---|
| 128080.SZ | 110.20 | 115.00 | -0.04 |
| 128083.SZ | 118.03 | 118 | 0.00 |
| 128081.SZ | 107.70 | 109.997 | -0.02 |
| 113550.SH | 105.45 | 108 | -0.02 |
| 123034.SZ | 106.09 | 103.1 | 0.03 |
| 110059.SH | 106.65 | 103.5 | 0.03 |
| 127014.SZ | 107.42 | 107 | 0.00 |
| 128078.SZ | 109.87 | 117.5 | -0.06 |
| 128077.SZ | 107.05 | 116.2 | -0.08 |
| 113545.SH | 107.06 | 101.2 | 0.06 |
| 128075.SZ | 107.82 | 105 | 0.03 |
| 113544.SH | 111.40 | 114.01 | -0.02 |
| 123032.SZ | 106.82 | 103.128 | 0.04 |
| 123030.SZ | 106.66 | 112.3 | -0.05 |
| 113543.SH | 124.91 | 123.64 | 0.01 |
| 113542.SH | 106.92 | 103.88 | 0.03 |
| 128069.SZ | 106.88 | 99.89 | 0.07 |
| 113536.SH | 106.38 | 102 | 0.04 |
| 113534.SH | 106.49 | 97 | 0.10 |
| 113024.SH | 107.34 | 103 | 0.04 |
| 128062.SZ | 110.15 | 108 | 0.02 |
| 110057.SH | 106.68 | 104 | 0.03 |
| 110056.SH | 107.35 | 109 | -0.02 |
| 123025.SZ | 112.70 | 112 | 0.01 |
| 113022.SH | 107.34 | 106 | 0.01 |
| 128059.SZ | 118.83 | 117 | 0.02 |
| 127011.SZ | 107.82 | 120 | -0.10 |
| 113528.SH | 106.86 | 113.5 | -0.06 |
| 110051.SH | 107.96 | 111.81 | -0.03 |
| 128056.SZ | 106.77 | 109 | -0.02 |
| Average deviation | | | 0.00078 |

Source: Authors' calculation.

## 4.4. B-S Model Analysis

### 4.4.1. Calculation of the value of call options

Then the option value can be obtained according to the B-S model formula.

**Table 6** Option Value Data

| Convertible bond code | Stock price (S) | Conversion price (X) | T-t | $d_1$ | $d_2$ | $N(d_1)$ | $N(d_2)$ | value of call options |
|---|---|---|---|---|---|---|---|---|
| 128080.SZ | 37.19 | 40.41 | 5.89 | 0.56 | 0.08 | 0.71 | 0.53 | 16.93 |
| 128083.SZ | 12.10 | 11.90 | 5.95 | 0.79 | 0.31 | 0.79 | 0.38 | 5.98 |
| 128081.SZ | 10.25 | 9.83 | 5.9 | 0.81 | 0.28 | 0.79 | 0.39 | 5.12 |
| 113550.SH | 13 | 9.93 | 5.89 | 1.02 | 0.07 | 0.85 | 0.47 | 7.34 |
| 123034.SZ | 7.7 | 7.98 | 5.89 | 0.75 | 0.37 | 0.77 | 0.36 | 3.73 |
| 110059.SH | 12.37 | 15.05 | 5.83 | 0.60 | 0.51 | 0.73 | 0.30 | 5.38 |
| 127014.SZ | 8.65 | 8.84 | 5.82 | 0.75 | 0.36 | 0.77 | 0.36 | 4.20 |
| 128078.SZ | 38.92 | 31.61 | 5.81 | 0.96 | 0.15 | 0.83 | 0.44 | 21.43 |
| 128077.SZ | 12.18 | 10.52 | 5.80 | 0.91 | 0.20 | 0.82 | 0.42 | 6.49 |
| 113545.SH | 10.79 | 11.4 | 5.79 | 0.72 | 0.38 | 0.76 | 0.35 | 5.12 |
| 128075.SZ | 6.01 | 5.79 | 5.73 | 0.80 | 0.30 | 0.79 | 0.38 | 2.99 |
| 113544.SH | 42.44 | 47.54 | 5.73 | 0.67 | 0.44 | 0.75 | 0.33 | 19.30 |
| 123032.SZ | 5.72 | 6.92 | 5.78 | 0.61 | 0.52 | 0.73 | 0.30 | 2.53 |
| 123030.SZ | 5.54 | 5.7 | 5.64 | 0.74 | 0.37 | 0.77 | 0.35 | 2.68 |
| 113543.SH | 117 | 101.46 | 5.63 | 0.88 | 0.16 | 0.81 | 0.44 | 59.97 |
| 113542.SH | 16.34 | 16.62 | 5.59 | 0.74 | 0.34 | 0.77 | 0.37 | 7.80 |
| 128069.SZ | 16.66 | 18.08 | 5.48 | 0.68 | 0.40 | 0.75 | 0.34 | 7.55 |
| 113536.SH | 20.02 | 19.75 | 5.42 | 0.76 | 0.31 | 0.78 | 0.38 | 9.59 |
| 113534.SH | 15.86 | 15.28 | 5.28 | 0.77 | 0.28 | 0.78 | 0.39 | 7.62 |
| 113024.SH | 7.13 | 9.87 | 5.27 | 0.43 | 0.63 | 0.67 | 0.27 | 2.65 |
| 128062.SZ | 6.43 | 16.25 | 5.26 | 0.14 | 1.20 | 0.44 | 0.12 | 1.34 |
| 110057.SH | 8.94 | 9.99 | 5.25 | 0.63 | 0.42 | 0.74 | 0.34 | 3.88 |
| 110056.SH | 16.26 | 15.68 | 5.22 | 0.77 | 0.28 | 0.78 | 0.39 | 7.76 |
| 123025.SZ | 54.84 | 50.25 | 5.25 | 0.82 | 0.24 | 0.79 | 0.41 | 27.10 |
| 113022.SH | 11.13 | 12.46 | 5.20 | 0.62 | 0.42 | 0.73 | 0.34 | 4.80 |
| 128059.SZ | 85.7 | 75.72 | 5.20 | 0.85 | 0.20 | 0.80 | 0.42 | 43.10 |
| 127011.SZ | 9.05 | 11.79 | 5.19 | 0.48 | 0.57 | 0.68 | 0.28 | 3.49 |
| 113528.SH | 19.25 | 24.03 | 5.17 | 0.52 | 0.53 | 0.70 | 0.30 | 7.65 |
| 110051.SH | 8.3 | 10.19 | 5.17 | 0.53 | 0.51 | 0.70 | 0.30 | 3.34 |
| 128056.SZ | 5.58 | 6.78 | 5.17 | 0.54 | 0.50 | 0.72 | 0.31 | 2.26 |

Source: WIND financial database and authors' calculation.

### *4.4.2. Convertible bond theoretical value*

**Table 7** B-S Model Results

| Convertible bond code | B-S model results | Actual market price | Deviation rate |
|---|---|---|---|
| 128080.SZ | 99.46 | 115.00 | -13.51% |
| 128083.SZ | 88.51 | 118 | -24.99% |
| 128081.SZ | 87.65 | 109.997 | -20.32% |
| 113550.SH | 89.87 | 108 | -16.79% |
| 123034.SZ | 86.26 | 103.1 | -16.33% |
| 110059.SH | 87.91 | 103.5 | -15.06% |
| 127014.SZ | 86.73 | 107 | -18.94% |
| 128078.SZ | 103.96 | 117.5 | -11.52% |
| 128077.SZ | 89.02 | 116.2 | -23.39% |
| 113545.SH | 87.65 | 101.2 | -13.39% |
| 128075.SZ | 85.52 | 105 | -18.55% |
| 113544.SH | 101.83 | 114.01 | -10.68% |
| 123032.SZ | 85.06 | 103.128 | -17.52% |
| 123030.SZ | 85.21 | 112.3 | -24.12% |
| 113543.SH | 142.50 | 123.64 | 15.25% |
| 113542.SH | 90.33 | 103.88 | -13.04% |
| 128069.SZ | 90.08 | 99.89 | -9.82% |
| 113536.SH | 92.12 | 102 | -9.69% |
| 113534.SH | 90.15 | 97 | -7.06% |
| 113024.SH | 85.18 | 103 | -17.30% |
| 128062.SZ | 83.87 | 108 | -22.34% |
| 110057.SH | 86.41 | 104 | -16.91% |
| 110056.SH | 90.29 | 109 | -17.17% |
| 123025.SZ | 109.63 | 112 | -2.12% |
| 113022.SH | 87.33 | 106 | -17.61% |
| 128059.SZ | 125.63 | 117 | 7.38% |
| 127011.SZ | 86.02 | 120 | -28.32% |
| 113528.SH | 90.18 | 113.5 | -20.55% |
| 110051.SH | 85.87 | 111.81 | -23.20% |
| 128056.SZ | 84.79 | 109 | -22.21% |
| Average deviation rate | | | -14.99% |

Source: Authors' calculation.

Table 7 shows that the theoretical prices are lower than the actual market prices for all the convertible bonds (except two bonds with positive value). The valuation formula is:

Convertible bond theoretical value = pure bond price + option value (7)

According to Figure 4, on the whole, when the original B-S model is used to calculate the pricing of China's convertible bonds, the deviation is still relatively large, with an average deviation rate of -14.99%. The actual market price is generally higher than the B-S model's price; this is consistent with Hui (2019) with the B-S model valuation of convertible bonds. According to Table 7, the theoretical prices are lower than the actual market prices for all the convertible bonds (except two bonds with positive values).

**Figure 4** Comparison of the B-S Model Price with Actual Market Price


### *4.5. CRR Model Analysis*

The parameters required to build the CRR model, such as interest rates and conversion prices, have been calculated in the B-S model.

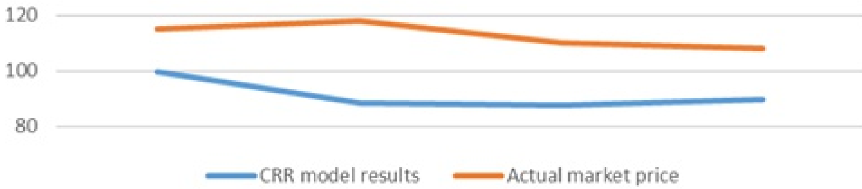
The critical parameters of the CRR model are the steps. Since the increase in the number of steps in the CRR model will significantly reduce the speed of the program, at the same time, to facilitate the calculation of convertible bond prices, we choose the number of steps in the CRR model to 9, which is a reasonable range of steps.

**Table 8** CRR Model Results

| Convertible bond code | CRR model results | Actual market price | Deviation rate |
|---|---|---|---|
| 128080.SZ | 99.71 | 115.00 | -13.30% |
| 128083.SZ | 88.52 | 118 | -24.99% |
| 128081.SZ | 87.66 | 109.997 | -20.31% |
| 113550.SH | 89.78 | 108 | -16.87% |
| 123034.SZ | 86.19 | 103.1 | -16.40% |
| 110059.SH | 87.74 | 103.5 | -15.23% |
| 127014.SZ | 86.73 | 107 | -18.94% |
| 128078.SZ | 103.81 | 117.5 | -11.65% |
| 128077.SZ | 89.00 | 116.2 | -23.41% |
| 113545.SH | 87.65 | 101.2 | -13.39% |
| 128075.SZ | 85.53 | 105 | -18.54% |
| 113544.SH | 101.76 | 114.01 | -10.74% |
| 123032.SZ | 84.98 | 103.128 | -17.59% |
| 123030.SZ | 85.17 | 112.3 | -24.16% |
| 113543.SH | 143.81 | 123.64 | 16.31% |
| 113542.SH | 90.34 | 103.88 | -13.04% |
| 128069.SZ | 90.07 | 99.89 | -9.83% |
| 113536.SH | 92.13 | 102 | -9.67% |
| 113534.SH | 90.16 | 97 | -7.05% |
| 113024.SH | 85.04 | 103 | -17.43% |
| 128062.SZ | 83.79 | 108 | -22.42% |
| 110057.SH | 86.40 | 104 | -16.92% |
| 110056.SH | 90.30 | 109 | -17.15% |
| 123025.SZ | 109.62 | 112 | -2.13% |
| 113022.SH | 87.31 | 106 | -17.63% |
| 128059.SZ | 125.54 | 117 | 7.30% |
| 127011.SZ | 85.91 | 120 | -28.41% |
| 113528.SH | 90.02 | 113.5 | -20.69% |
| 110051.SH | 85.81 | 111.81 | -23.25% |
| 128056.SZ | 84.76 | 109 | -22.24% |
| Average deviation | | | -14.99% |

Source: Authors' calculation.

**Figure 5** Comparison of the CRR Model Price with Actual Market Price




It can be seen from Table 8 and Figure 5 that the convertible bond price of the CRR model also has a deviation from the actual market price, and the deviation is significant. It is worth noting that the deviation rate is consistent with the B-S model's deviation rate, which is -14.99%.

It can be seen from the results in Table 8 that the CRR model and the B-S model are also consistent. Their theoretical prices are lower than the actual market price for the rest of the convertible bonds. This shows that the value of the most convertible bonds selected from the current market sample is overestimated.

### *4.6. Monte Carlo Simulation Analysis*

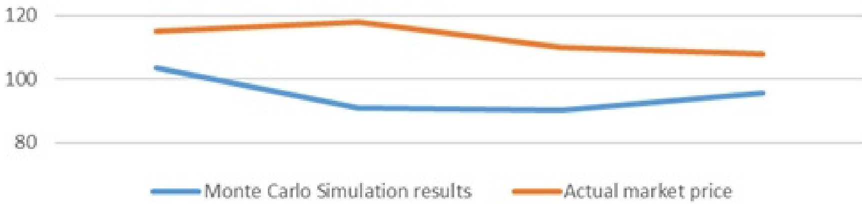
Compared with the B-S and CRR models, the Monte Carlo simulation approach is more flexible, more likely to be assumed, and has more extensive application fields. The shortcoming of the Monte Carlo simulation approach is that the amount of calculation is too large. Therefore, to simplify the simulation process and facilitate calculation, but at the same time, to ensure the accuracy of the simulated price, this paper chooses 360 simulation paths to determine the price of convertible bonds. Moreover, the data used in the Monte Carlo simulation are consistent with the B-S model and the CRR model.

**Table 9** Monte Carlo Simulation Results

| Convertible bond code | Monte Carlo Simulation results | Actual market price | Deviation rate |
|---|---|---|---|
| 128080.SZ | 103.65 | 115.00 | -9.87% |
| 128083.SZ | 90.78 | 118 | -23.07% |
| 128081.SZ | 90.25 | 109.997 | -17.95% |
| 113550.SH | 95.63 | 108 | -11.45% |
| 123034.SZ | 88.12 | 103.1 | -14.53% |
| 110059.SH | 92.67 | 103.5 | -10.46% |
| 127014.SZ | 89.55 | 107 | -16.31% |
| 128078.SZ | 102.79 | 117.5 | -12.52% |
| 128077.SZ | 95.8 | 116.2 | -17.56% |
| 113545.SH | 95.2 | 101.2 | -5.93% |
| 128075.SZ | 91.73 | 105 | -12.64% |
| 113544.SH | 109.41 | 114.01 | -4.03% |
| 123032.SZ | 88.58 | 103.128 | -14.11% |
| 123030.SZ | 86.81 | 112.3 | -22.70% |
| 113543.SH | 164.85 | 123.64 | 33.33% |
| 113542.SH | 94.54 | 103.88 | -8.99% |
| 128069.SZ | 96.13 | 99.89 | -3.76% |
| 113536.SH | 95.87 | 102 | -6.01% |
| 113534.SH | 100.53 | 97 | 3.64% |
| 113024.SH | 86.22 | 103 | -16.29% |
| 128062.SZ | 87.71 | 108 | -18.79% |
| 110057.SH | 91.51 | 104 | -12.01% |
| 110056.SH | 96.51 | 109 | -11.46% |
| 123025.SZ | 146.24 | 112 | 30.57% |
| 113022.SH | 96.06 | 106 | -9.38% |
| 128059.SZ | 136.83 | 117 | 16.95% |
| 127011.SZ | 94.27 | 120 | -21.44% |
| 113528.SH | 93.99 | 113.5 | -17.19% |
| 110051.SH | 91.02 | 111.81 | -18.59% |
| 128056.SZ | 87.02 | 109 | -20.17% |
| Average deviation rate | | | -9.09% |

Source: Authors' calculation.

**Figure 6** Comparison of Monte Carlo Simulation Price with Actual Market Price




According to Figure 6, the Monte Carlo simulation results are generally the same as a B-S model and CRR model, both of which are theoretical prices lower than actual market prices, which shows that the model set in this article is correct.

At the same time, it can be seen from Table 9 that the deviation rate of the Monte Carlo simulation results is -9.09%. This deviation rate is lower than the deviation rate of the B-S model and the CRR model. Indeed, if more paths are simulated, the deviation rate will be lower.

### *4.7. Reasons for the Deviation*

There has not been much research on the systematic factors that affect the deviation of convertible bonds. King (1986) believes that maturity is an essential factor. The degree of underestimation is generally the most serious on the first day of issuance. As the remaining maturity decreases, the degree of underestimation also gradually decreases.

Based on a sample of US convertible bonds, Hillion and Vermaelen (2004) believe that the existing valuation models rarely consider the call options embedded in convertible bonds, which will lead to convertible bonds to a significant value deviation. When analyzing the causes of underestimation and considering its default risk, it should also focus on designing additional terms of convertible bonds, especially the option value of the resale clause and redemption clause.

Numerous scholars have studied the price deviation between the convertible bond's theoretical model and actual market price from empirical data (Broomhead and King, 1986; Kang and Lee, 1996). Besides, Hutchinson and Gallagher (2005) conducted a study on French convertible bonds from 19 February 1999 to 5 September 2000, using a binary tree model, concluded that the average theoretical value of French convertible bonds is underestimated by 3 per cent.

In a Rotaru (2006) on Japanese convertible bonds, the author uses the CIR stochastic interest rate model with a 100-step trigeminal tree model to conduct a valuation analysis. The results show that most of the 35 convertible bonds are overvalued, and the market price is on average 1.7 per cent higher than the theoretical price calculated by the model.

The deviation between the theoretical and actual prices has been laid out based on empirical findings and the discussion above. The first reason is that the model does not consider any additional terms, such as redemption, resale, and conversion. The embedded options also need to be priced reasonably. The stepwise regression model, B-S model, CRR model and Monte Carlo simulation do not consider the impact of these terms on the valuation of convertible bonds (Zabolotnyuk, Jones and Veld, 2010).

Based on the above literature, it could be concluded that similar reasons also occur in China's convertible bond market. The deviation between the actual and theoretical price is that valuation models do not consider embedded call options, sell-back terms and redemption terms.

## 5. Conclusion

Table 10 compares the stepwise regression model, B-S model, CRR model and Monte Carlo simulation. The results show that the stepwise regression model has the lowest deviation rate of 0.00078, followed by Monte Carlo simulation at -9.09%. The B-S and CRR models have the most significant deviation rate of -14.99%.

This result shows that the stepwise regression model results have the best fit, and the B-S model and the CRR model have the worst fit. Our first conclusion is that stepwise Regression and Monte Carlo simulation are more suitable for China's convertible bond market.

**Table 10** Summary of Deviation Rate from Different Models

| Stepwise regression model | B-S model | CRR model | Monte Carlo Simulation |
|---|---|---|---|
| 0.00078 | -14.99% | -14.99% | -9.09% |

Source: Authors' calculation.

**Figure 7** Comparison of Four Model Prices with Actual Market Price

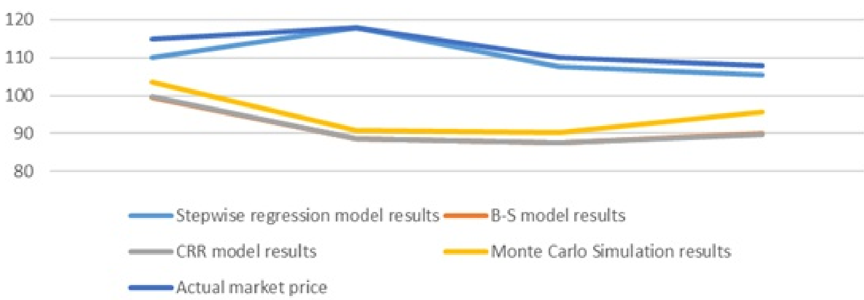


Figure 7 shows that all models have a lower price than the actual market price. There is sufficient evidence to show that the convertible bonds selected for our sample are overestimated in the current market. Our second conclusion is that maturity is an essential factor affecting convertible bonds' value from the above figure. It is the most underestimated on the first day of issuance. However, the degree of underestimation gradually diminishes as the life span of the bond decreases over time.

As a policy implication, the price discrepancies between theoretical and actual prices should be monitored continuously by the regulators and market practitioners in China's financial markets. The valuation of convertible bonds has paramount importance to bond investors, issuers, and market regulators. Investors can combine the price information of convertible bonds with the trend of the company's stock and information on the macro-financial environment (interest rates and regular bond prices) for choosing which convertible bonds to invest in. For financial regulators, accurate pricing models and timely information enable them to objectively evaluate the market operation and maintain a sound financial system by establishing a bond pricing system that provides a formulation for improving the regulatory rules.

## Notes

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1. Refer to <*https://www.wind.com.cn/en/edb.html*>, accessed on 2nd January 2021.

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